EE 230
Lecture 19

Nonlinear Circuits
Nonlinear Op Amp Applications

Review from Last Time:

Types of Nonlinearities

- Continuously differentiable transfer characteristic

$$\frac{d^k f}{dx^k} \text{ exists for all or some } k \geq = 1 \text{ and all } x$$

- Piecewise Transfer Characteristic

functional form of f_1 and f_2 differ

$$Y = \begin{cases} f_1(x) & x < x_1 \\ f_2(x) & x \geq x_1 \end{cases}$$

$$f_1(x_1) = f_2(x_1)$$

1

- Piecewise and continuously differentiable

- Discontinuous

- Multi-valued

- Many other types

Review from Last Time:

Analysis of nonlinear circuits is often much more difficult than analysis of linear circuits

Analysis of nonlinear circuits is often much easier than analysis of linear circuits

Some very useful circuits are nonlinear circuits
- Almost all logic circuits
- ADCs and DACs

Most semiconductor devices are nonlinear at the most basic level
- MOSFET, BJT, Diode, …

Often a large number of nonlinear devices are combined to form a linear (or nearly linear) circuit

Often a large number of nonlinear devices are combined to form higher-level nonlinear circuits that are very useful and much easier to analyze than the constituent devices

Nonlinear circuits are very widely used and analysis techniques for these circuits must be developed

Methods of Analysis of Nonlinear Circuits

KCL and KVL apply to both linear and nonlinear circuits

Superposition, voltage divider and current divider equations, Thevenin and Norton equivalence apply only to linear circuits!

Some other analysis techniques that have been developed may apply only to linear circuits as well

Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course

1. Nonlinear circuits with continuously differential devices

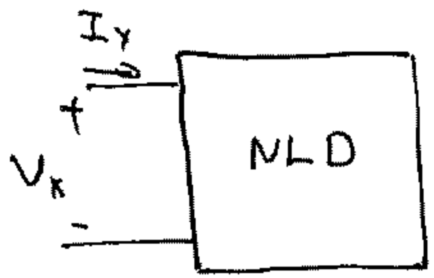
Analysis Strategy:

Use KVL and KCL for analysis

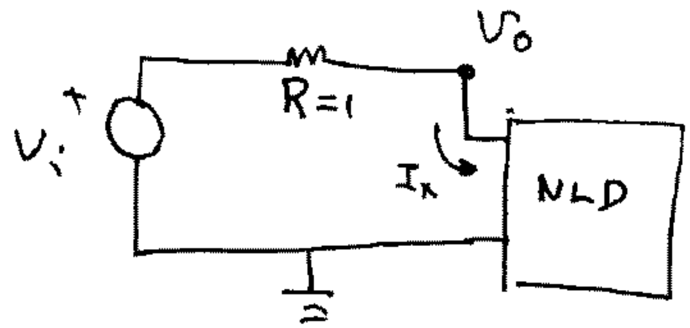
Represent nonlinear models for devices either mathematically or graphically

Solve the resultant set of equations for the variables of interest

Example



$$I_x = 3e^{V_x}$$



Goal: Obtain V_o

$$\left.\begin{aligned} \frac{V_i - V_o}{R} &= I_x \\ I_x &= 3e^{V_o} \end{aligned}\right\}$$

$$\frac{V_i - V_o}{R} = 3e^{V_o}$$

$$V_{in} = V_o + 3Re^{V_o}$$

$$V_{in} = V_o + 3e^{V_o}$$

- Solution relating V_o to V_{in} is highly nonlinear
- Explicit expression for V_o not obtainable for this simple nonlinear circuit

- Solution may be very involved

- Explicit expression for V_o or V_{in} or both is often impossible to obtain

- Most useful nonlinear circuits will have reasonably simple final expressions for output variable of interest and a systematic procedure for analyzing these circuits

2. Circuits with piecewise continuous devices

e.g. $$f(x) = \begin{cases} f_1(x) & x < x_1 \quad \text{region 1} \\ f_2(x) & x > x_1 \quad \text{region 2} \end{cases}$$

Analysis Strategy:

Guess region of operation

Solve resultant circuit using the previous method

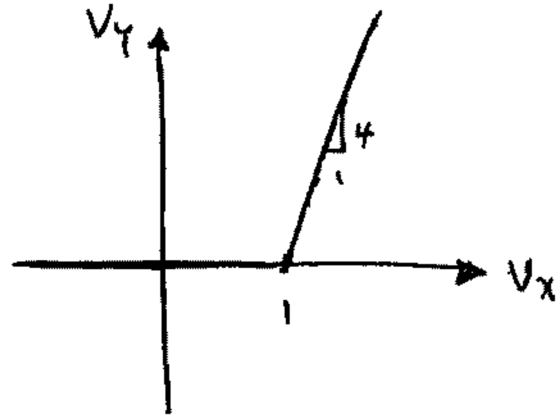
Verify region of operation is valid

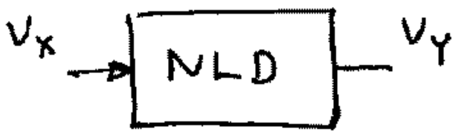
Repeat the previous 3 steps as often as necessary until region of operation is verified

It helps to guess right the first time but a wrong guess will not result in an incorrect solution because a wrong guess can not be verified

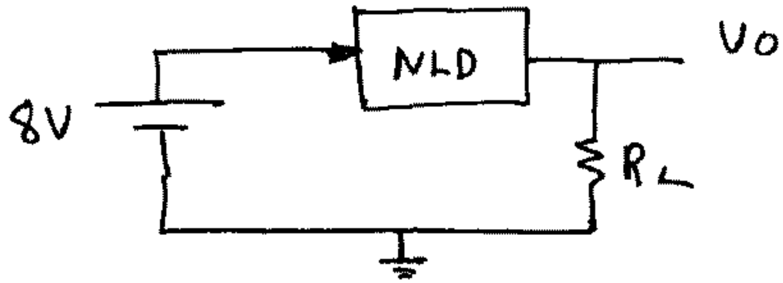
Example:

$$V_Y = \begin{cases} 0 & V_X < 1 \quad \text{"1"} \\ 4V_X - 4 & V_X > 1 \quad \text{"2"} \end{cases}$$





Circuit with NLD



Analysis:

1. Guess Region "1".

$V_O = V_Y = 0$

to verify

$V_X = 8V, \quad V_X \not< 1.$

$\therefore$ Solution not valid

2. Guess Region "2"

$V_O = (4)(8) - 4 = 28V$

to verify

$V_X = 8V, \quad V_X > 1$

$\therefore$ Solution is valid

$$\boxed{V_O = 28V}$$

3. Circuits with small-signal inputs that vary around some operating point

Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Analysis Strategy:

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0)

Develop small signal (linear) model for all devices in the region of interest (around the operating point or “Q-point”)

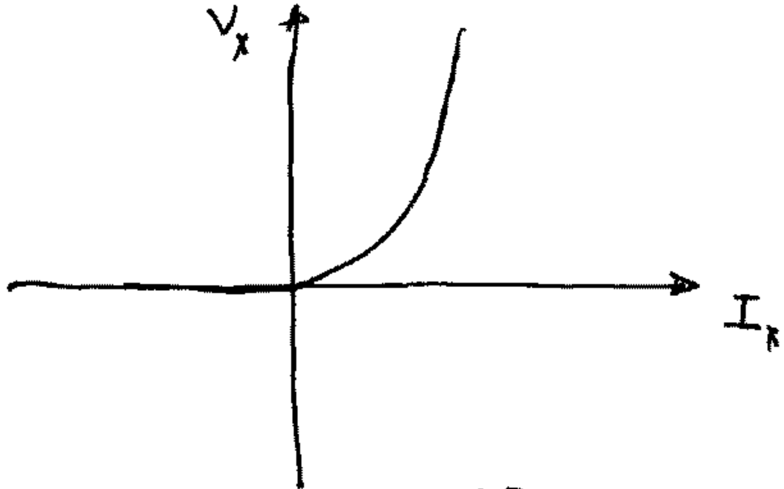
Create small signal equivalent circuit by replacing <u>all</u> devices with small-signal equivalent

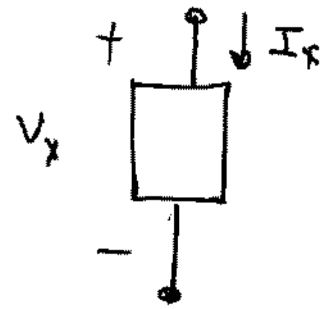
Solve the resultant small-signal (linear) circuit

Can use KCL, DVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence

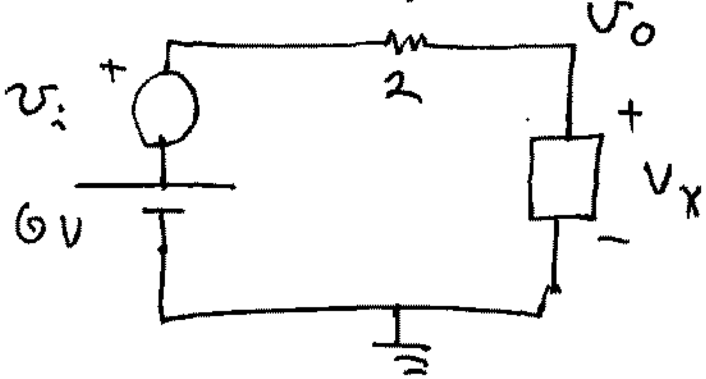
Determine boundary of region where small signal analysis is valid

Example:





$$V_x = \begin{cases} I_x^2 & I_x > 0 \\ 0 & I_x < 0 \end{cases}$$

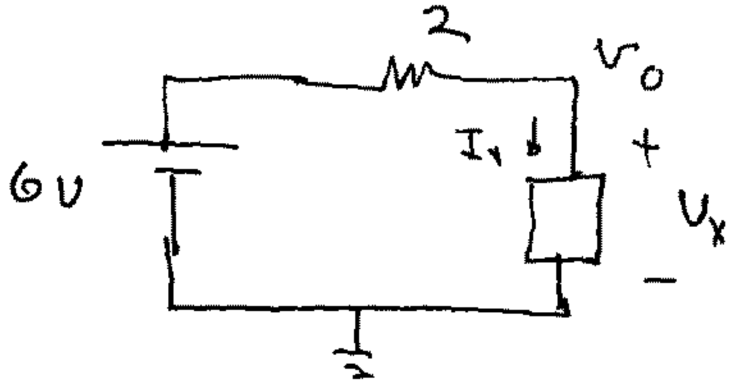


Assume v_i is small

Obtain v_o for small chances in v_i

(maybe $v_i = V_m \sin \omega t$)

To obtain operating point, let $v_i = 0$



$$V_x = \begin{cases} I_x^2 & I_x > 0 \\ 0 & I_x < 0 \end{cases}$$

Guess $I_x > 0$

$$\left.\begin{aligned} I_x &= \frac{6 - V_o}{2} \\ V_x &= I_x^2 \\ V_x &= V_o \end{aligned}\right\} \qquad V_o = \left(\frac{6 - V_o}{2}\right)^2$$

To be completed during next lecture